Recall that composition was introduced as the result of substituting one polynomial function, \( f \), into another, \( g \). The result is another polynomial function, \( f \circ g \).

**Definition of a Composite Function**

The composite function \( f \circ g \) is defined by \((f \circ g)(x) = f(g(x))\).

The notation \( f \circ g \) is read “\( f \) follows \( g \)” or “the composition of \( f \) and \( g \).”

To evaluate \( f(g(x)) \), first evaluate \( g(x) \) and then evaluate \( f \) at the number \( g(x) \).

The input-output diagram shows the composite function \( f \circ g \).

To determine the output of \( f \circ g \), use the output of \( g \) as the input of \( f \).

You can think of a composite function in this way: a change in one quantity produces a change in a second quantity, which in turn produces a change in a third quantity. For example, in many Northern Ontario communities, the human population affects the wolf population, which in turn affects the rabbit population. When the human population increases, the local wolf population decreases, because the wolves tend to avoid humans. The decrease in the wolf population leads to an increase in the rabbit population, since rabbits are eaten by wolves. What would happen to the rabbit population if the human population decreased?

This population example illustrates the idea of composition. However, other factors influence the populations of humans, wolves, and rabbits.

Can you use composition to combine functions that are not polynomials? Can you combine a function and its inverse? Do the properties for polynomial composite functions apply to other composite functions as well? In this section, you will extend your understanding of composition to include polynomial, rational, inverse, and other functions.
**EXAMINING THE CONCEPT**

**Composition Involving Functions Represented by Discrete Data**

Before combining, or composing, different types of functions, you should understand some basic properties of composite functions.

### Example 1

**Composition of Functions Defined by Tables**

Functions \( f \) and \( g \) are defined in the tables on the left. Create a table for \( f \circ g \).

#### Solution

For \( f \circ g \), start with the table for \( g \). Recall the input-output diagram. The values for \( x \) are the inputs. The outputs are the values for \( g(x) \). These outputs become the inputs for \( f \). So the second column of the first table becomes the first column for the second table shown here. Now treat each value for \( g(x) \) as an input for \( f \). Find the values for \( f(g(x)) \), if they exist, in the second table below. For example,

\[
\begin{align*}
  x &\rightarrow g(x) & \rightarrow f(g(x)) \\
  2 &\rightarrow 0 & \rightarrow 3 \\
  4 &\rightarrow -3 & \rightarrow \text{undefined}
\end{align*}
\]

To obtain the table for \( f \circ g \), choose the rows in the other tables where \( x \), \( g(x) \), and \( f(g(x)) \) are all defined.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( g(x) )</th>
<th>( f(g(x)) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>2.5</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-3</td>
<td>undefined</td>
</tr>
</tbody>
</table>

### The Domain and Range of \( f \circ g \)

The composition of two functions exists only where the range of the first function overlaps, or is contained in, the domain of the second function. The domain of \( f \circ g \) is a subset of the domain of \( g \). The range of \( f \circ g \) is a subset of the range of \( f \).

What happens when a function is composed with its inverse? Recall that the inverse function “undoes” the effect of the original function. As a result, the domain of the original function becomes the range of the inverse. The range of the original function becomes the domain of the inverse.
Example 2  Composition of a Discrete Function and Its Inverse

For function \( g \) in Example 1, find \( g \circ g^{-1} \) and \( g^{-1} \circ g \).

**Solution**

Rewrite \( g \) as a set of ordered pairs.

\[
g = \{(-1, 2.5), (0, 3), (1, -2), (2, 0), (3, 1), (4, -3)\}
\]

Therefore, 

\[
g^{-1} = \{(2.5, -1), (3, 0), (-2, 1), (0, 2), (1, 3), (-3, 4)\}
\]

\[
g^{-1} \circ g = \{(-1, -1), (0, 0), (1, 1), (2, 2), (3, 3), (4, 4)\}
\]

\[
g \circ g^{-1} = \{(2.5, 2.5), (3, 3), (-2, -2), (0, 0), (1, 1), (-3, -3)\}
\]

In this example, \((g^{-1} \circ g)(x) = x\) for all \(x\) in the domain of \(g\) and \((g \circ g^{-1})(x) = x\) for all \(x\) in the domain of \(g^{-1}\).

The next example shows that this pattern is not always true.

Example 3  Composition of Discrete Relations

The arrow diagrams on the right represent relations \(k\) and \(k^{-1}\). Draw an arrow diagram for \(k^{-1} \circ k\). Explain why \((k^{-1} \circ k)(x) = x\) is not true for all \(x\) in the domain of \(k\). What condition would guarantee that, in general, \((f^{-1} \circ f)(x) = x\) for all \(x\) in the domain of \(f\)?

**Solution**

The arrow diagram for \(k^{-1} \circ k\) is shown on the left. Note that \((0, 0) \in k^{-1} \circ k\) and \((0, 1) \in k^{-1} \circ k\), so the relation is not a function.

So, \((k^{-1} \circ k)(x) \neq x\) for all \(x\) in the domain of \(k\). Note that while \(k\) is a function, \(k^{-1}\) is not.

The Composition of a Function and Its Inverse

If both \(f\) and \(f^{-1}\) are functions, then

- \((f^{-1} \circ f)(x) = x\) for all \(x\) in the domain of \(f\), and
- \((f \circ f^{-1})(x) = x\) for all \(x\) in the domain of \(f^{-1}\).

EXAMINING THE CONCEPT

Algebraic Composition Involving Functions

The examples so far have shown some properties of composite functions defined by ordered pairs or discrete data. In section 1.3, you composed algebraically a polynomial function with another polynomial function. Now extend composition to include other types of functions that are defined algebraically.
Algebraic Composition of Functions

Let \( f(x) = 2x + 3 \) and \( g(x) = \frac{5x + 1}{2x - 3} \). Determine \((g \circ f)(x)\) and \((f \circ g)(x)\).

**Solution**

By definition, \((g \circ f)(x) = g(f(x))\).

\[
(g \circ f)(x) = g(2x + 3) = \frac{5(2x + 3) + 1}{2(2x + 3) - 3} = \frac{10x + 16}{4x + 3}
\]

By definition, \((f \circ g)(x) = f(g(x))\).

\[
(f \circ g)(x) = f\left(\frac{5x + 1}{2x - 3}\right) = 2\left(\frac{5x + 1}{2x - 3}\right) + 3 = \frac{10x + 2 + 6x - 9}{2x - 3} = \frac{16x - 7}{2x - 3}
\]

Example 4 shows an important relationship to remember when working with composite functions: You may often need to restrict the domain of a composite function to ensure that it is defined.

**Example 5**

**The Domain and Range of a Composite Function**

Let \( h(x) = 3x + 12 \) and \( k(x) = \sqrt{x} \). What are the domain and range of \( k \circ h \)?

**Solution**

Substitute the expression for \( h(x) \) into the expression for \( k \).

\[
(k \circ h)(x) = k(h(x)) = k(3x + 12) = \sqrt{3x + 12}
\]

The expression for \( k \circ h \) is defined only if \( 3x + 12 \geq 0 \) or \( x \geq -4 \). The value of the expression can be any real number \( y \) such that \( y \geq 0 \). Therefore, the domain of \( k \circ h \) is \( \{x \mid x \geq -4, x \in \mathbb{R}\} \). The range is \( \{y \mid y \geq 0, y \in \mathbb{R}\} \).

Note: In this example, \( h \) is applied first. Since the domain of \( h \) is \( \mathbb{R} \), you might think that the domain of \( k \circ h \) is also \( \mathbb{R} \). But since the output from \( h \) is also the input to \( k \), there are restrictions on the domain of the composite function.

Sometimes it is useful to know the actual functions that have been combined to create a composite function. The process of determining these functions is called **decomposition**.
**Example 6  Decomposing a Composite Function**

Let \( h(x) = (x^2 + 6)^5 - 9 \). Find two functions \( f \) and \( g \) such that \( h(x) = f(g(x)) \).

**Solution**

Think of applying one function (the inner function) first and applying the other (the outer function) next. To evaluate \( h(x) \), you would first evaluate \( x^2 + 6 \). So choose \( g(x) = x^2 + 6 \) as the inner function. The next step is to raise \( u = g(x) \) to the power 5 and subtract 9, so take \( f(u) = u^5 - 9 \) as the outer function.

With \( g(x) = x^2 + 6 \) and \( f(x) = x^5 - 9 \), then

\[
\begin{align*}
(f \circ g)(x) &= f(u) = (x^2 + 6)^5 - 9 \\
&= (x^2 + 6)^5 - 9.
\end{align*}
\]

Note that this is just one of many possible solutions.

**Example 7  Composition of a Function and Its Inverse**

Given \( h(x) = 2x - 3 \), determine \( h^{-1}(x) \), \( (h \circ h^{-1})(x) \), and \( (h^{-1} \circ h)(x) \).

**Solution**

Recall that you can find the inverse function by switching \( x \) and \( y \) and solving for \( y \). Thus, \( y = 2x - 3 \) becomes \( x = 2y - 3 \). Solving \( x = 2y - 3 \) for \( y \) gives \( y = \frac{x + 3}{2} \). Therefore, \( h^{-1}(x) = \frac{x + 3}{2} \).

Now find \( (h \circ h^{-1})(x) \) and \( (h^{-1} \circ h)(x) \).

\[
\begin{align*}
(h \circ h^{-1})(x) &= h(h^{-1}(x)) \\
&= h\left(\frac{x + 3}{2}\right) \\
&= 2\left(\frac{x + 3}{2}\right) - 3 \\
&= x
\end{align*}
\]

\[
\begin{align*}
(h^{-1} \circ h)(x) &= h^{-1}(h(x)) \\
&= h^{-1}(2x - 3) \\
&= \frac{(2x - 3) + 3}{2} \\
&= x
\end{align*}
\]

This result seems logical for any function \( f \) and its inverse, \( f^{-1} \). Since the inverse function “undoes” the original function, the composition \( f^{-1} \circ f \) maps the domain of \( f \) onto the range of \( f \), then back onto the domain of \( f \). The net result of composing a function with its inverse function (or vice versa) is to map the domain of the original function onto itself.

---

The function \( f(x) = x \) is called the identity function.

---

The Composition of a Function and Its Inverse

As you saw earlier, provided that both \( f \) and \( f^{-1} \) are functions,

\[
(f^{-1} \circ f)(x) = (f \circ f^{-1})(x) = x.
\]
CHECK, CONSOLIDATE, COMMUNICATE

1. Why is it reasonable that \( f \circ g \) is read “\( f \) follows \( g \)?
2. When do points in \( g \circ f \) exist? Explain.
3. Why is \( (a, a) \in f \circ g \) if \( (a, b) \in f \)?
4. Let \( f(x) = \sqrt{x} \) and \( g(x) = x + 1 \). Why is \( \{ x \mid x \geq -1, x \in \mathbb{R} \} \) the domain of \( f \circ g \)?

KEY IDEAS

• \( f \circ g \) is the **composite** function of \( f \) and \( g \). The composite function is defined by \( (f \circ g)(x) = f(g(x)) \). To determine \( (f \circ g)(x) \), replace \( x \) with \( g(x) \) in the expression for \( f(x) \).

• Let \( (a, b) \in g \) and \( (b, c) \in f \). Then \( (a, c) \in f \circ g \). A point in \( f \circ g \) exists where an element in the range of \( g \) is also in the domain of \( f \). The function \( f \circ g \) exists only when the range of \( g \) overlaps the domain of \( f \).

• The domain of \( f \circ g \) is a subset of the domain of \( g \). Find the domain of \( f \circ g \) by examining \( f \circ g \) and comparing the domain of \( f \) with the range of \( g \).

• If \( (a, b) \in f \), then \( (b, a) \in f^{-1} \). So, \( (a, a) \in f^{-1} \circ f \) and \( (b, b) \in f \circ f^{-1} \).

• If both \( f \) and \( f^{-1} \) are functions, then \( (f^{-1} \circ f)(x) = x \) for all \( x \) in the domain of \( f \), and \( (f \circ f^{-1})(x) = x \) for all \( x \) in the domain of \( f^{-1} \).

6.1 Exercises

A 1. (a) Let \( f = \{(0, 1), (1, 3), (2, 5), (3, 7)\} \) and \( g(x) = 2x + 1 \). Evaluate
   i. \( (g \circ f)(0) \)  ii. \( (g \circ f)(1) \)  iii. \( (g \circ f)(2) \)  iv. \( (g \circ f)(3) \)
   v. \( (g \circ f)(4) \)  vi. \( (f \circ g)(0) \)  vii. \( (f \circ g)(1) \)  viii. \( (f \circ g)(2) \)

   (b) Determine the domain of \( f \circ g \).
   (c) Graph \( y = (f \circ g)(x) \).
   (d) Determine the domain of \( g \circ f \).
   (e) Graph \( y = (g \circ f)(x) \).
2. (a) Let \( f(x) = 3x - 1 \) and \( g(x) = 5 - 2x \). Determine
   
   i. \((f \circ g)(x)\)  
   ii. \((f \circ g)(0)\)  
   iii. \((f \circ g)(-1)\)  
   iv. \((f \circ g)(2)\)  
   v. \((g \circ f)(x)\)  
   vi. \((g \circ f)(0)\)  
   vii. \((g \circ f)(-1)\)  
   viii. \((g \circ f)(2)\)

(b) Solve \((f \circ g)(x) = (g \circ f)(x)\).

3. Let \( f = \{ (1, 2), (2, 3), (3, 5), (4, 7) \} \) and \( g = \{ (1, 4), (2, 3), (3, 1) \} \).
   Express each composite as a set of ordered pairs.
   
   (a) \( g \circ f \)
   (b) \( f \circ g \)
   (c) \( f^{-1} \circ f \)
   (d) \( g \circ g^{-1} \)

4. (a) Let \( f(x) = x^2 \) and \( g(x) = x + 2 \). Determine
   
   i. \((f \circ g)(x)\)  
   ii. \((g \circ f)(x)\)

(b) Solve \((f \circ g)(x) = (g \circ f)(x)\).

5. Let \( f(x) = 3x - 5 \). Show that \((f \circ f^{-1})(x) = x\).

6. Knowledge and Understanding: Let \( f(x) = x^2 - 3x \) and \( g(x) = \sqrt{x} \).
   Determine
   
   (a) \((f \circ g)(5)\)  
   (b) \((g \circ f)(x)\)  
   (c) the domain of \( g \circ f \)

7. For each function \( h \), find two functions \( f \) and \( g \) such that \( h(x) = f(g(x)) \).
   
   (a) \( h(x) = \sqrt{x^2 + 6} \)  
   (b) \( h(x) = (5x - 8)^6 \)
   
   (c) \( h(x) = 2^{(6x + 7)} \)  
   (d) \( h(x) = \frac{1}{x^3 - 7x + 2} \)
   
   (e) \( h(x) = \sin^2(10x + 5) \)  
   (f) \( h(x) = \sqrt{x + 4)^2} \)

8. Copy and complete the table.

<table>
<thead>
<tr>
<th>Point on ( f )</th>
<th>Point on ( g )</th>
<th>Point on ( f \circ g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3, 4)</td>
<td>(3, 1)</td>
<td></td>
</tr>
<tr>
<td>(1, 5)</td>
<td>(2, 1)</td>
<td></td>
</tr>
<tr>
<td>(0, -1)</td>
<td>(1, ■)</td>
<td></td>
</tr>
<tr>
<td>(■, 5)</td>
<td>(■, 1)</td>
<td>(0, ■)</td>
</tr>
<tr>
<td>(3, ■)</td>
<td></td>
<td>(-1, 3)</td>
</tr>
</tbody>
</table>

9. Given the graphs of \( f \) and \( g \),
   
   (a) graph \( f \circ g \)
   (b) graph \( g \circ f \)
10. (a) Let \( f = \{(−2, 1), (0, 4), (1, 2), (4, 5)\} \) and \( g(x) = \sqrt{x − 3} \). Evaluate
   i. \((f \circ g)(3)\)     ii. \((f \circ g)(4)\)     iii. \((f \circ g)(7)\)
(b) Determine the domain of \( f \circ g \).
(c) Graph \( f \circ g \).
(d) Evaluate
   i. \((g \circ f)(0)\)     ii. \((g \circ f)(−2)\)     iii. \((g \circ f)(7)\)
(e) Determine the domain of \( g \circ f \).
(f) Graph \( g \circ f \).

11. Let \( f(x) = 2x + 3 \) and \( g(x) = ax − 2 \).
   (a) Create a table for \( f \) if \(-2 \leq x \leq 2, x \in \mathbb{I} \), and \( a = 3 \).
   (b) Add columns for \( g \) and \( g \circ f \) to your table.
   (c) Graph \( f \), \( g \), and \( g \circ f \). Highlight the point on each graph that
       corresponds to the point on the graph of \( f \) where \( x = −2 \).
   (d) Repeat (a) to (c) for \( a = 2 \), \( a = −1 \), and \( a = −3 \).
   (e) Describe the effect of changing the value of \( a \) on the graph of \( g \circ f \).

12. Communication:

   (a) Given the graphs of \( f \) and \( g \), graph \( f \circ g \).
   (b) Graph \( g \circ f \).
   (c) How do your graphs show that \( f \) and \( g \) are not inverses of each other?

13. Given the graph of \( y = f(x) \) on the right and the functions \( g(x) = 2x + 1 \),
   \( h(x) = −x + 3 \), and \( k(x) = (g \circ f \circ h)(x) \),
   (a) evaluate
      i. \( k(5) \)     ii. \( k(7) \)     iii. \( k(3) \)
      iv. \( k(−2) \)     v. \( k(0) \)     vi. \( k(6) \)
   (b) graph \( y = k(x) \)
   (c) explain how transforming the graph of \( y = f(x) \) can create the graph of \( y = k(x) \)
14. Given the graph of \( y = f(x) \) on the right and the functions below, match the correct composition with each graph. Justify your choices.

\[
g(x) = x + 3 \quad h(x) = 0.5x \quad k(x) = -x \\
m(x) = 2x \quad n(x) = -0.5x \quad p(x) = x - 3
\]

(a) \( y = f \circ g(x) \) \quad (b) \( y = f \circ h(x) \) \quad (c) \( y = f \circ k(x) \)

(d) \( y = f \circ m(x) \) \quad (e) \( y = f \circ n(x) \) \quad (f) \( y = f \circ p(x) \)

(g) \( y = g \circ f(x) \) \quad (h) \( y = h \circ f(x) \) \quad (i) \( y = k \circ f(x) \)

(j) \( y = m \circ f(x) \) \quad (k) \( y = n \circ f(x) \) \quad (l) \( y = p \circ f(x) \)
15. (a) Let \( f(x) = 2x - 1 \) and \( g(x) = x^2 \). Determine \( (f \circ g)(x) \).
(b) Graph \( f \), \( g \), and \( f \circ g \) on the same set of axes.
(c) Describe the graph of \( f \circ g \) as a transformation of the graph of \( y = g(x) \).

16. (a) Let \( f(x) = 2x - 1 \) and \( g(x) = 3x + 2 \). Determine \( (f \circ g)(x) \).
(b) Graph \( f \), \( g \), and \( f \circ g \) on the same set of axes.
(c) Draw the input-output diagram for \( f \circ g \).
(d) Describe the graph of \( f \circ g \) as a transformation of the graph of \( y = g(x) \).
(e) Describe the graph of \( f \circ g \) as a transformation of the graph of \( y = f(x) \).

17. Let \( f(x) = 2(x - 1) \), \( g(x) = x^2 \), and \( h(x) = \sqrt{3x + 2} \). Find the domain and range of \( h \circ g \circ f \).

18. Let \( f(x) = ax^2 + bx + c \) and \( g(x) = mx + n \), where \( a, b, c, m, n \in \mathbb{R} \).
(a) Determine \( (f \circ g)(x) \).
(b) Determine \( (g \circ f)(x) \).
(c) What can you conclude about the composition of a linear function with a quadratic function?

19. For each pair of functions, find \( f(g(x)) \) and \( g(f(x)) \).
(a) \( f(x) = 1 - x^2 \) and \( g(x) = 2x + 5 \)
(b) \( f(x) = 5x \) and \( g(x) = \sqrt{4x + 2} \)
(c) \( f(x) = \sqrt{x^2 - 2} \) and \( g(x) = x^2 + 2 \)
(d) \( f(x) = x^2 + 1 \) and \( g(x) = \frac{1}{x} \)
(e) \( f(x) = x^3 - 4 \) and \( g(x) = \sqrt{x + 4} \)
(f) \( f(x) = \sin x \) and \( g(x) = x^2 + 2 \)
(g) \( f(x) = 2x^2 + 3x - 2 \) and \( g(x) = x^\frac{1}{2} \)
(h) \( f(x) = \frac{2x - 1}{5x} \) and \( g(x) = x^2 \)

20. Let \( f(x) = x - 3 \). Determine
(a) \( (f \circ f)(x) \)
(b) \( (f \circ f \circ f)(x) \)
(c) \( (f \circ f \circ f \circ f)(x) \)
(d) \( f \) composed with itself \( n \) times

21. **Thinking, Inquiry, Problem Solving:** Let \( f(x) = x^2 - 3x \). Determine \( g(x) \) so that \( (f \circ g)(x) = x^2 + x - 2 \).

22. **Check Your Understanding:** Show that the composition of any two linear functions is a linear function.
23. Nadia collected the data in the table on the left using a CBR after a ball was dropped.

(a) Enter the data into \( L1 \) and \( L2 \) using a TI-83 Plus. Graph the relation.

(b) Without looking at the graph, how do you know that the relation is not linear?

(c) Use regression to find the equation of the quadratic of best fit.

(d) Is this equation a good model? Justify your answer.

You can use the TI-83 Plus to estimate different regression models, but often the model does not perfectly fit the data. A common method in data analysis is to transform one variable until the graph becomes linear. In this case, observe that time increases by a constant, but height decreases by a larger amount in each successive observation. This suggests a transformation that compresses height or expands time.

(e) Calculate the square root of each entry in \( L2 \) (height) and store the values in \( L3 \).

(f) Graph the square root of height versus time. Does this relation appear to be linear? Calculate the correlation coefficient for the relation. What can you conclude?

(g) Calculate the square of each time-value in \( L1 \) and store the results in \( L3 \).

(h) Graph height versus time\(^2 \). Does this relation appear to be linear? Calculate the correlation coefficient. What can you conclude?

(i) Let \( h \) represent height and \( t \) represent time. Express \( h(t) \) as the composition of two other functions.

24. **Application**: The number of bicycles, \( n \), sold at one store in a week is a function of the price, \( p \), in dollars. So \( n(p) = \frac{5(360 - p)}{p - 80} \) for \( p > 80 \).

The store’s cost, \( c \), in dollars for each bike is a function of the number of bikes the store sells each week. So \( c(n) = 0.002(n + 2)^2 + 80 \).

(a) Evaluate \( n(100) \) and \( n(180) \). Why are these values reasonable in this situation?

(b) Evaluate \( c(8) \) and \( c(48) \). Why are these values reasonable in this situation?

(c) Evaluate the cost of each bicycle to the store if the selling price is $120.

(d) Determine the store’s profit per bicycle if the selling price is $120.

(e) Evaluate the total profit if the selling price is $120.

(f) Express the cost of each bike to the store as a function of the selling price.

(g) Express the total profit in terms of the functions \( c \) and \( n \) and the variable \( p \).

(h) Use graphing technology to graph total profit versus price.
**ADDITIONAL ACHIEVEMENT CHART QUESTIONS**

**Knowledge and Understanding:** Given \( f(x) = \frac{x + 3}{2} \) and \( g(x) = \sqrt{-4x + 1} \), find

(a) \((f \circ g)(-2)\)  \hspace{1cm} (b) the value of \( x \) such that \((g \circ f)(x) = 10\)

(c) the domain of \((f \circ g)(x)\)  \hspace{1cm} (d) the range of \((g \circ f)(x)\)

**Application:** A banquet hall charges $975 to rent a reception room, plus $25.95 per person. This month, the hall is offering a discount of 30% off the total bill. Express the discounted cost as a function of the number of people attending a reception.

**Thinking, Inquiry, Problem Solving:** The function \( f(x) = (2x + 3)^2 \) is the composition of two functions, \( g(x) \) and \( h(x) \). Find at least two different pairs of functions \( g(x) \) and \( h(x) \) such that \( f(x) = (g \circ h)(x) \).

**Communication:** Send an e-mail message to a classmate explaining how to find the domain of \((f \circ g)(x)\), where \( f(x) = 5x + 7 \) and \( g(x) = \sqrt{x} \). Then, to check that person’s understanding, ask your classmate to reply with an explanation of how to find the domain of \((g \circ f)(x)\).

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**The Chapter Problem**

**Tides on the Bay of Fundy**

Apply what you learned in this section to answer these questions about The Chapter Problem on page 430.

**CP1.** Convert the times in the tide table on page 430 into decimal numbers. Graph the depth of water in the bay versus time. Draw a smooth curve through the points.

**CP2.** Use the graph to estimate the time and rate at which the water level was falling when it was falling most quickly.

**CP3.** Assume that the floor of the bay slopes at a constant ratio of 1:20. Express the distance the water flows up the beach in terms of depth.

**CP4.** The depth of water in the bay is a function of time. The distance the water flows up the beach is a function of the depth. Express the distance the water flows up the beach as a composition of functions.